Higher Institute for Engineering& Modern Technology

Preparatory Year

Lecture Notes on

Electromagnetic Fields

For First Year

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Higher Institute for Engineering& Modern Technology
Electrical Engineering Department

Lecture Notes on

Electromagnetic Fields

For First Year

MIE

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Chapter 1: Vector Analysis

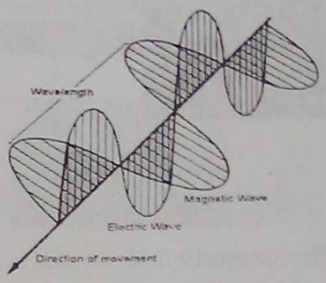
What is Electromagnetics?

Electric field

Produced by the presence of electrically charged particles, and gives rise to the electric force.

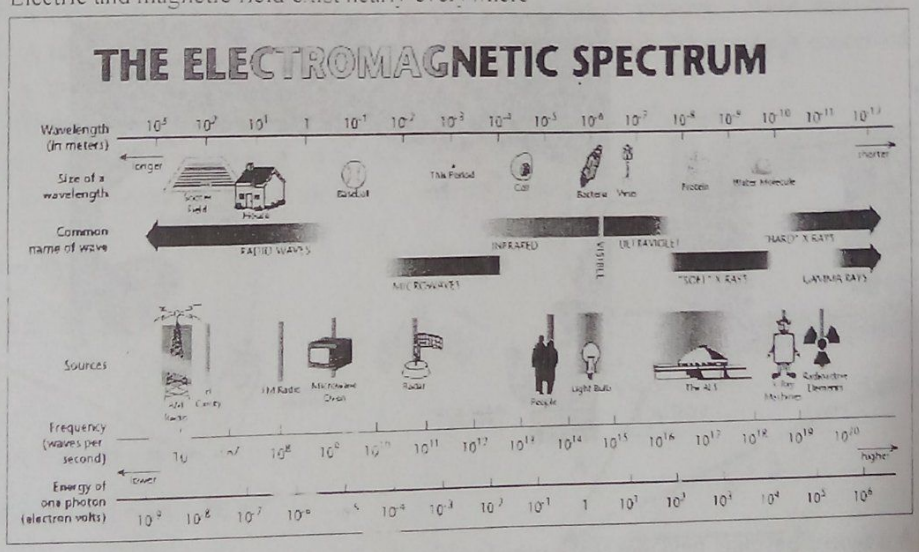
Magnetic field

Produced by the mo-tion of electric charges, or electric current, and gives rise to the magnetic force associated with magnets.



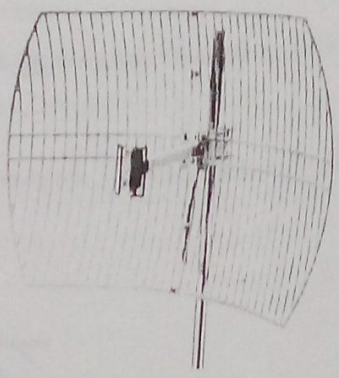
Why do we learn Engineering Electromagnetics

- Electric and magnetic field exist nearly everywhere



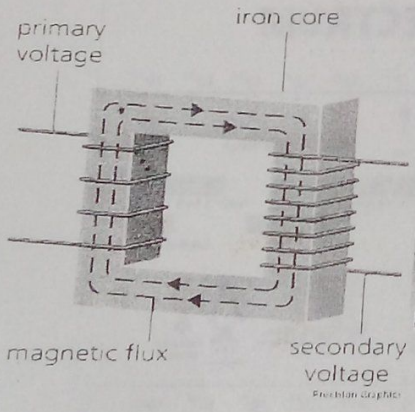
- Electromagnetic principles find application in various disciplines such as microwaves, x-rays, antennas, electric machines, plasmas, etc.

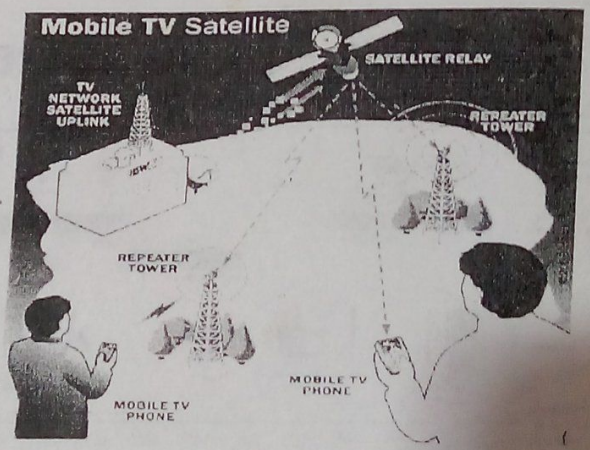




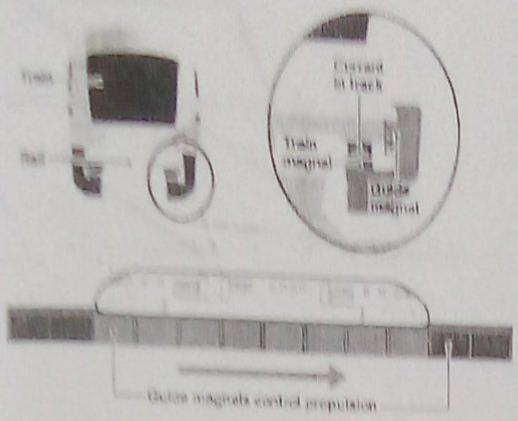


- Electromagnetic fields are used in induction heaters for melting, forging, annealing, surface hardening, and soldering operation.
- Electromagnetic devices include transformers, radio, television, mobile phones, radars, lasers, etc.

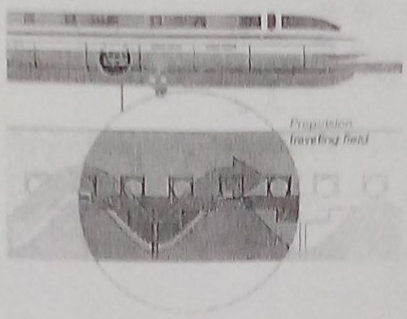




- A magnetic traveling field moves the vehicle without contact.
- The speed can be continuously regulated by varying the frequency of the alternating current.







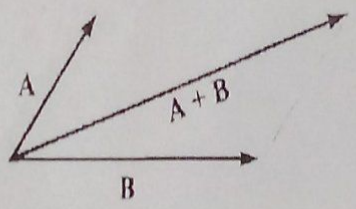
- Scalar refers to a quantity whose value may be represented by a single (positive or negative) real number.

- Some examples include distance, temperature, mass, density, pressure, volume, and time.
- A vector quantity has both a magnitude and a direction in space. We especially concerned with two- and three-dimensional spaces only.
- Displacement, velocity, acceleration, and force are examples of vectors.
  - Scalar notation: A or A (*italic* or plain)
  - Vector notation: $\mathbf{A}$ or $\vec{A}$ (**bold** or plain with arrow)

$A + B = B + A$

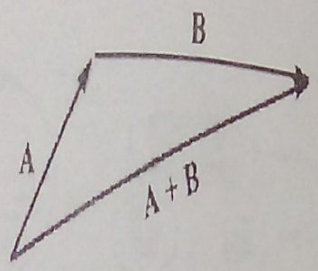
$A + (B + C) = (A + B) + C$

$A - B = A + (-B)$



$$\frac{A}{n} = \frac{1}{n}A$$

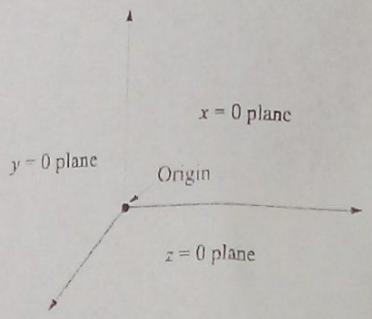
$$A - B = 0 \rightarrow A = B$$



Rectangular Coordinate System

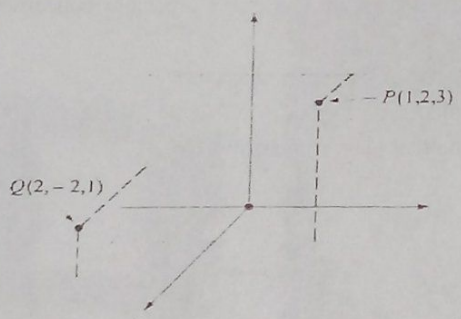
- Differential surface units:

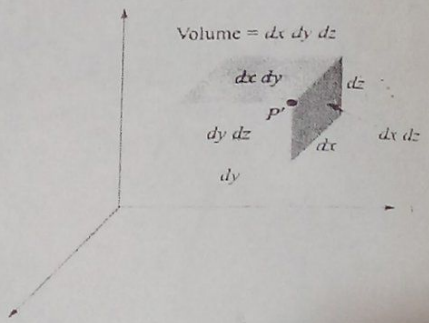
  $dx \cdot dy$
  $dy \cdot dz$
  $dx \cdot dz$



- Differential volume unit :

  $dx \cdot dy \cdot dz$



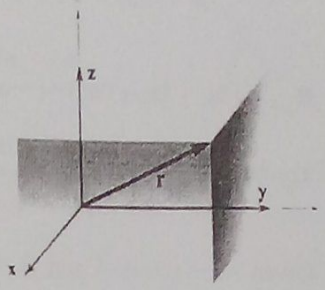


Vector Components and Unit Vectors

$\mathbf{r} = \mathbf{x} + \mathbf{y} + \mathbf{z}$

$\mathbf{r} = x\mathbf{a}_x + y\mathbf{a}_y + z\mathbf{a}_z$

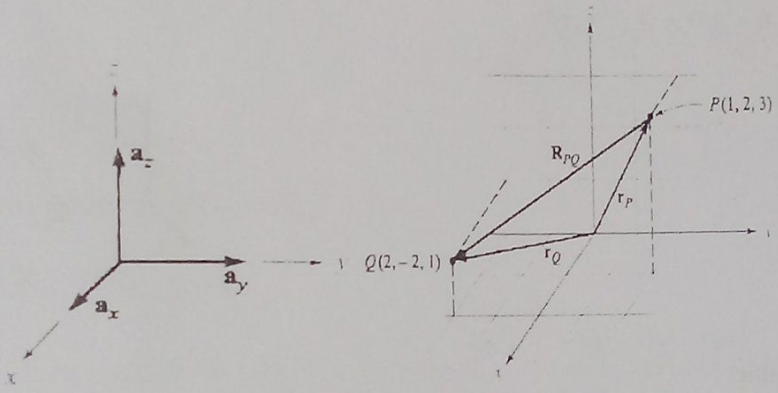
$\mathbf{a}_x, \mathbf{a}_y, \mathbf{a}_z$ unit vectors



$$\mathbf{R}_{PQ} = \mathbf{r}_Q - \mathbf{r}_P$$

$$= (2\mathbf{a}_x - 2\mathbf{a}_y + \mathbf{a}_z) - (1\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)$$

$$= \mathbf{a}_x - 4\mathbf{a}_y - 2\mathbf{a}_z$$



- For any vector $\mathbf{B}$: $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

$$|\mathbf{B}| = \sqrt{B_x^2 + B_y^2 + B_z^2} = B$$ Magnitude of $\mathbf{B}$

$$\mathbf{a}_B = \frac{\mathbf{B}}{\sqrt{B_x^2 + B_y^2 + B_z^2}} = \frac{\mathbf{B}}{|\mathbf{B}|}$$ Unit vector in the direction of $\mathbf{B}$

Example

Given points $M(-1,2,1)$ and $N(3,-3,0)$, find R_{MN} and a_{MN}.

$$\mathbf{R}_{MN} = (3\mathbf{a}_x - 3\mathbf{a}_y + 0\mathbf{a}_z) - (-1\mathbf{a}_x + 2\mathbf{a}_y + 1\mathbf{a}_z) = \underline{4\mathbf{a}_x - 5\mathbf{a}_y - \mathbf{a}_z}$$

$$\mathbf{a}_{MN} = \frac{\mathbf{R}_{MN}}{|\mathbf{R}_{MN}|} = \frac{4\mathbf{a}_x - 5\mathbf{a}_y - 1\mathbf{a}_z}{\sqrt{4^2 + (-5)^2 + (-1)^2}} = \underline{0.617\mathbf{a}_x - 0.772\mathbf{a}_y - 0.154\mathbf{a}_z}$$

- Given two vectors A and B, the *dot product*, or *scalar product*, is defines as the product of the magnitude of A, the magnitude of B, and the cosine of the smaller angle between them

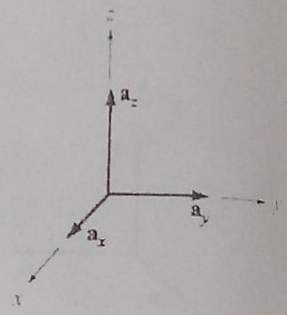
$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos\theta_{AB}$$

- The dot product is a scalar, and it obeys the commutative law:

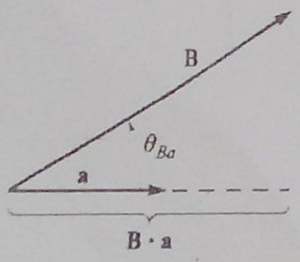
$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

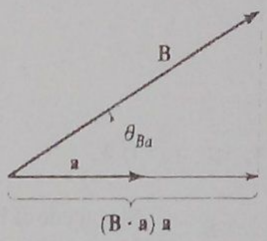
For any vector $\mathbf{A} = A_x\mathbf{a}_x + A_y\mathbf{a}_y + A_z\mathbf{a}_z$ and $\mathbf{B} = B_x\mathbf{a}_x + B_y\mathbf{a}_y + B_z\mathbf{a}_z$

$$\mathbf{A} \cdot \mathbf{B} = A_xB_x + A_yB_y + A_zB_z$$



The Dot Product





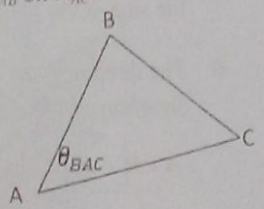
- The scalar component of **B** in the direction of the unit vector **a** is **B**·**a**
- The vector component of **B** in the direction of the unit vector **a** is (**B**·**a**)**a**

$$\mathbf{B} \cdot \mathbf{a} = |\mathbf{B}||\mathbf{a}|\cos\theta_{Ba} = |\mathbf{B}|\cos\theta_{Ba}$$

Example

The three vertices of a triangle are located at $A(6,-1,2)$, $B(-2,3,-4)$, and $C(-3,1,5)$. Find: (*a*) $\mathbf{R}_{AB}$; (*b*) $\mathbf{R}_{AC}$; (*c*) the angle θ_{BAC} at vertex A; (*d*) the vector projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.

$$\mathbf{R}_{AB} = (-2\mathbf{a}_x + 3\mathbf{a}_y - 4\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z}}$$



$$\mathbf{R}_{AC} = (-3\mathbf{a}_x + 1\mathbf{a}_y + 5\mathbf{a}_z) - (6\mathbf{a}_x - \mathbf{a}_y + 2\mathbf{a}_z) = \underline{\underline{-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z}}$$

$$\mathbf{R}_{AB} \cdot \mathbf{R}_{AC} = |\mathbf{R}_{AB}||\mathbf{R}_{AC}|\cos\theta_{BAC}$$

$$\Rightarrow \cos\theta_{BAC} = \frac{\mathbf{R}_{AB} \cdot \mathbf{R}_{AC}}{|\mathbf{R}_{AB}||\mathbf{R}_{AC}|} = \frac{(-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z) \cdot (-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-8)^2 + (4)^2 + (-6)^2}\right|\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|} = \frac{62}{\left|\sqrt{116}\right|\left|\sqrt{94}\right|} = 0.594$$

$$\Rightarrow \theta_{BAC} = \cos^{-1}(0.594) = \underline{\underline{53.56^\circ}}$$

Example

The three vertices of a triangle are located at $A(6,-1,2)$, $B(-2,3,-4)$, and $C(-3,1,5)$. Find:

(*a*) $\mathbf{R}_{AB}$;

(*b*) $\mathbf{R}_{AC}$;

(*c*) the angle θ_{BAC} at vertex A;

(*d*) the vector projection of $\mathbf{R}_{AB}$ on $\mathbf{R}_{AC}$.

$$\mathbf{R}_{AB} \text{ on } \mathbf{R}_{AC} = (\mathbf{R}_{AB} \cdot \mathbf{a}_{AC})\mathbf{a}_{AC}$$

$$= \left((-8\mathbf{a}_x + 4\mathbf{a}_y - 6\mathbf{a}_z) \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|} \right) \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\left|\sqrt{(-9)^2 + (2)^2 + (3)^2}\right|}$$

$$= \frac{62}{\sqrt{94}} \frac{(-9\mathbf{a}_x + 2\mathbf{a}_y + 3\mathbf{a}_z)}{\sqrt{94}}$$

$$= \underline{\underline{-5.963\mathbf{a}_x + 1.319\mathbf{a}_y + 1.979\mathbf{a}_z}}$$

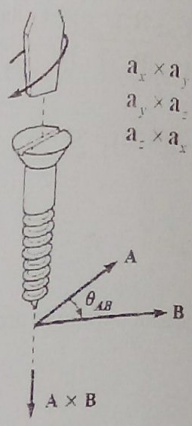
The Cross Product

- Given two vectors **A** and **B**, the magnitude of the *cross product*, or *vector product*, written as $\mathbf{A}\times\mathbf{B}$, is defines as the product of the magnitude of **A**, the magnitude of **B**, and the sine of the smaller angle between them.
- The direction of $\mathbf{A}\times\mathbf{B}$ is perpendicular to the plane containing **A** and **B** and is in the direction of advance of a right-handed screw as **A** is turned into **B**.

$$\mathbf{A}\times\mathbf{B} = \mathbf{a}_N |\mathbf{A}||\mathbf{B}|\sin\theta_{AB}$$

- The cross product is a vector, and it is not commutative.

$$(\mathbf{B}\times\mathbf{A}) = -(\mathbf{A}\times\mathbf{B})$$



Example

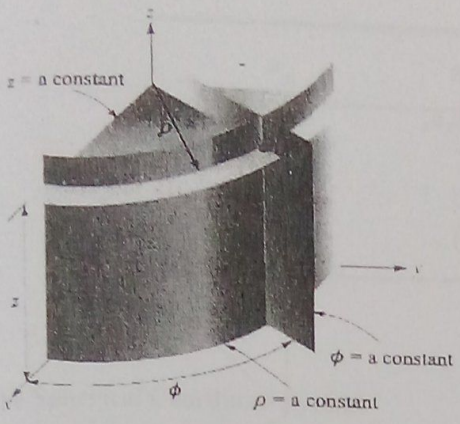
Given $\mathbf{A} = 2\mathbf{a}_x - 3\mathbf{a}_y + \mathbf{a}_z$ and $\mathbf{B} = -4\mathbf{a}_x - 2\mathbf{a}_y + 5\mathbf{a}_z$, find $\mathbf{A}\times\mathbf{B}$.

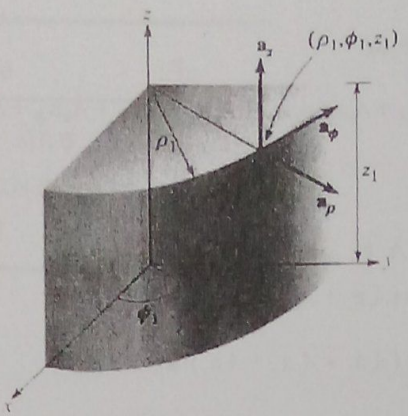
$$\mathbf{A}\times\mathbf{B} = (A_yB_z - A_zB_y)\mathbf{a}_x + (A_zB_x - A_xB_z)\mathbf{a}_y + (A_xB_y - A_yB_x)\mathbf{a}_z$$

$$= \big((-3)(5) - (1)(-2)\big)\mathbf{a}_x + \big((1)(-4) - (2)(5)\big)\mathbf{a}_y + \big((2)(-2) - (-3)(-4)\big)\mathbf{a}_z$$

$$= \underline{\underline{-13\mathbf{a}_x - 14\mathbf{a}_y - 16\mathbf{a}_z}}$$

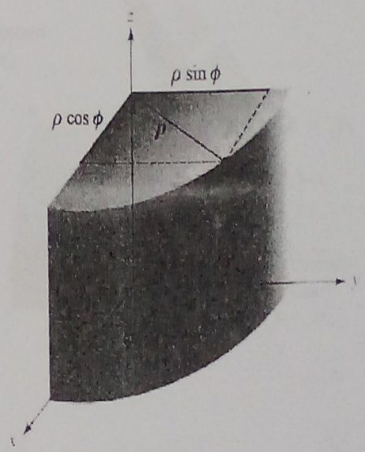
The Cylindrical Coordinate System





- Relation between the rectangular and the cylindrical coordinate systems

$x = \rho \cdot \cos\phi$	$\rho = \sqrt{x^2 + y^2}$
$y = \rho \cdot \sin\phi$	$\phi = \tan^{-1}\frac{y}{x}$
$z = z$	$z = z$

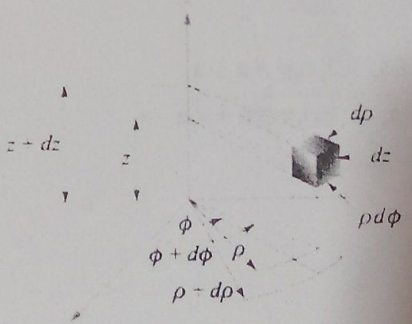


- Differential surface units:

$d\rho \cdot dz$

$\rho d\phi \cdot dz$

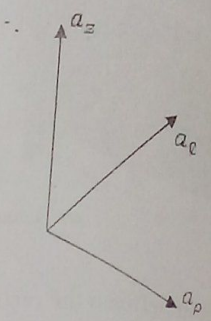
$d\rho \cdot \rho d\phi$



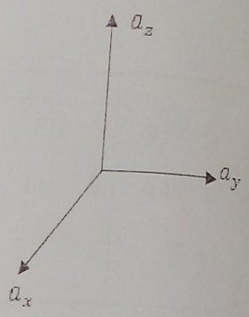
- Differential volume unit :

$d\rho \cdot \rho d\phi \cdot dz$

$$\mathbf{A} = A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z \rightarrow \mathbf{A} = A_\rho \mathbf{a}_\rho + A_\phi \mathbf{a}_\phi + A_z \mathbf{a}_z$$



$$A_\rho = \mathbf{A} \cdot \mathbf{a}_\rho$$
$$= (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_\rho$$
$$= (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_\rho$$
$$= A_x \cos\phi + A_y \sin\phi$$



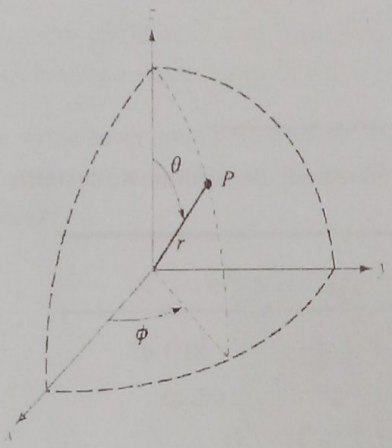
$$A_\phi = \mathbf{A} \cdot \mathbf{a}_\phi$$
$$= (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_\phi$$
$$= A_x \mathbf{a}_x \cdot \mathbf{a}_\phi + A_y \mathbf{a}_y \cdot \mathbf{a}_\phi + A_z \mathbf{a}_z \cdot \mathbf{a}_\phi$$
$$= -A_x \sin\phi + A_y \cos\phi$$

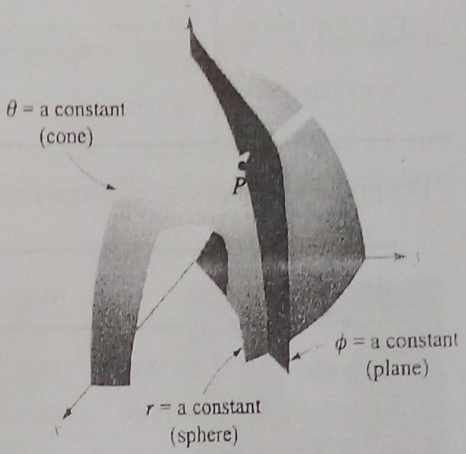
$$A_z = \mathbf{A} \cdot \mathbf{a}_z$$
$$= (A_x \mathbf{a}_x + A_y \mathbf{a}_y + A_z \mathbf{a}_z) \cdot \mathbf{a}_z$$
$$= A_x \mathbf{a}_x \cdot \mathbf{a}_z + A_y \mathbf{a}_y \cdot \mathbf{a}_z + A_z \mathbf{a}_z \cdot \mathbf{a}_z$$
$$= A_z$$

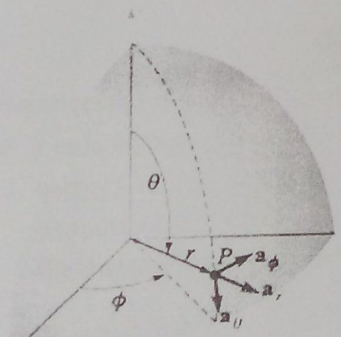
- Dot products of unit vectors in cylindrical and rectangular coordinate systems

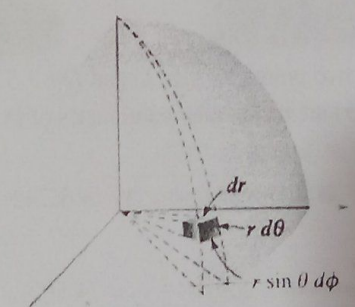
	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x$ ·	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y$ ·	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z$ ·	0	0	1

The Spherical Coordinate System





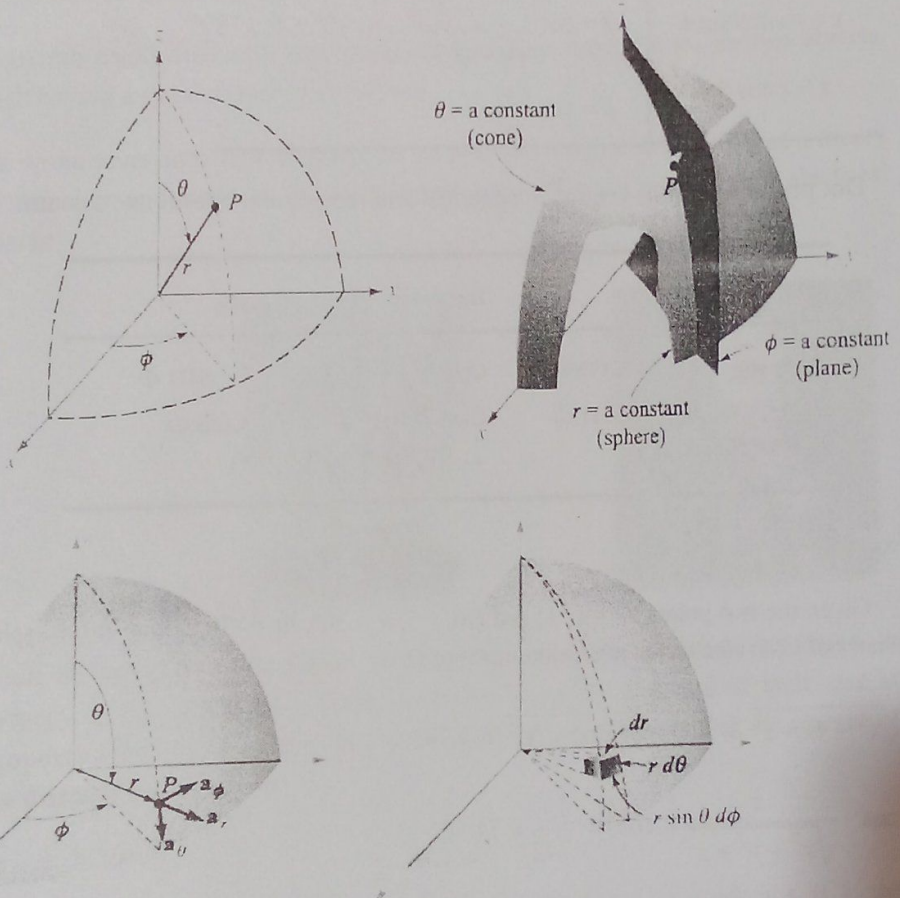




- Dot products of unit vectors in cylindrical and rectangular coordinate systems

	$\mathbf{a}_\rho$	$\mathbf{a}_\phi$	$\mathbf{a}_z$
$\mathbf{a}_x$	$\cos\phi$	$-\sin\phi$	0
$\mathbf{a}_y$	$\sin\phi$	$\cos\phi$	0
$\mathbf{a}_z$	0	0	1

The Spherical Coordinate System



- Differential surface units:

$dr \cdot rd\theta$

$dr \cdot r\sin\theta d\phi$

$rd\theta \cdot r\sin\theta d\phi$

- Differential volume unit

$dr \cdot rd\theta \cdot r\sin\theta d\phi$

- Relation between the rectangular and the spherical coordinate systems

$x = r\sin\theta\cos\phi$	$r = \sqrt{x^2 + y^2 + z^2}, \quad r \geq 0$
$y = r\sin\theta\sin\phi$	$\theta = \cos^{-1}\dfrac{z}{\sqrt{x^2 + y^2 + z^2}}, \quad 0° \leq \theta \leq 180°$
$z = r\cos\theta$	$\phi = \tan^{-1}\dfrac{y}{x}$

- Dot products of unit vectors in spherical and rectangular coordinate systems

	$\mathbf{a}_r$	$\mathbf{a}_\theta$	$\mathbf{a}_\phi$
$\mathbf{a}_x$ ·	$\sin\theta\cos\phi$	$\cos\theta\cos\phi$	$-\sin\phi$
$\mathbf{a}_y$ ·	$\sin\theta\sin\phi$	$\cos\theta\sin\phi$	$\cos\phi$
$\mathbf{a}_z$ ·	$\cos\theta$	$-\sin\theta$	0

Example

Given the two points, $C(-3,2,1)$ and $D(r = 5, \theta = 20°, \Phi = -70°)$, find: (a) the spherical coordinates of C; (b) the rectangular coordinates of D.

$$r = \sqrt{x^2 + y^2 + z^2} = \sqrt{(-3)^2 + (2)^2 + (1)^2} = 3.742$$

$$\theta = \cos^{-1}\frac{z}{\sqrt{x^2 + y^2 + z^2}} = \cos^{-1}\frac{1}{3.742} = 74.50°$$

$$\phi = \tan^{-1}\frac{y}{x} = \tan^{-1}\frac{2}{-3} = -33.69° + 180° = 146.31°$$

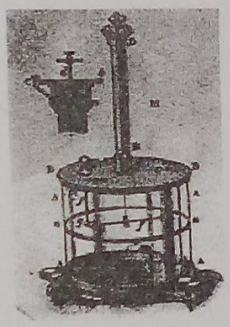
$$\therefore C(r = 3.742, \theta = 74.50°, \phi = 146.31°)$$

$$\therefore D(x = 0.585, y = -1.607, z = 4.698)$$

Chapter 2: Coulomb's Law and Electric Field Intensity

The Experimental Law of Coulomb

- In 1600, Dr. Gilbert, a physician from England, published the first major classification of electric and non-electric materials.
- He stated that glass, sulfur, amber, and some other materials "not only draw to themselves straw, and chaff, but all metals, wood, leaves, stone, earths, even water and oil."
- In the following century, a French Army Engineer, Colonel Charles Coulomb, performed an elaborate series of experiments using devices invented by himself.
- Coulomb could determine quantitatively the force exerted between two objects, each having a static charge of electricity.
- He wrote seven important treatises on electric and magnetism, developed a theory of attraction and repulsion between bodies of the opposite and the same electrical charge.





- Coulomb stated that the **force** between two very small objects separated in vacuum or free space by a distance which is large compared to their size is **proportional** to the **charge** on each and **inversely proportional** to the **square of the distance** between them

$$F = k\frac{Q_1 Q_2}{R^2}$$

- In SI Units, the quantities of charge Q are measured in coulombs (C), the separation R in meters (m), and the force F should be newtons (N).
- This will be achieved if the constant of proportionality k is written as:

$$k = \frac{1}{4\pi\varepsilon_0}$$